State Examination Commission – Physics Higher Level, 2007

Question 6

State Hooke's law. (6)
A stretched spring obeys Hooke's law. When a small sphere of mass 300 g is attached to a spring of length 200 mm, its length increases to 285 mm.
Calculate its spring constant. (9)
The sphere is pulled down until the length of the spring is 310 mm. The sphere is then released and oscillates about a fixed point. Derive the relationship between the acceleration of the sphere and its displacement from the fixed point. Why does the sphere oscillate with simple harmonic motion? (18)
Calculate:
(i) the period of oscillation of the sphere
(ii) the maximum acceleration of the sphere
(iii) the length of the spring when the acceleration of the sphere is zero. (23)
(acceleration due to gravity = 9.8 m s^{-2})

State Hooke's law. (6)

Basic statement

A stretched spring obeys Hooke's law. When a small sphere of mass 300 g is attached to a spring of length 200 mm, its length increases to 285 mm.
Calculate its spring constant. (9)

$F = mg = ks$
$0.30 \times 9.8 = k \times 0.085$
$k = 34.6 \text{ N m}^{-1}$

The sphere is pulled down until the length of the spring is 310 mm. The sphere is then released and oscillates about a fixed point.
Derive the relationship between the acceleration of the sphere and its displacement from the fixed point. Why does the sphere oscillate with simple harmonic motion? (18)

$F = ma$
$ma = -ks$
$\Rightarrow a = -(k/m)s = -\omega^2 s$ where, $\omega^2 = k/m$ *(constant of proportionality)*
or $a \propto -s$

The sphere oscillates with SHM as its acceleration is proportional to its displacement from a fixed point and is in the direction of that point.

Calculate:
(i) the period of oscillation of the sphere

$T = 2\pi/\omega = 2\pi/\sqrt{(k/m)} = 2\pi/\sqrt{(34.6/0.300)} = 2\pi/\sqrt{115.3}$
$= 0.6 \text{ s}$

(ii) the maximum acceleration of the sphere

$a = -\omega^2 s = -115.3 \times 0.025 = -2.89 \text{ ms}^{-2}$

(iii) the length of the spring when the acceleration of the sphere is zero. (23)

The acceleration is zero when the sphere is undisplaced from its equilibium position, i.e., when the spring is 285 mm long.

(acceleration due to gravity = 9.8 m s^{-2})