Electricity
Some Laws, Definitions and Formulae

Coulomb's Law : The force between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance separating them,

$$F \propto \frac{Q_1 Q_2}{d^2} \qquad \text{or} \qquad F = \frac{1}{4\pi\varepsilon} \cdot \frac{Q_1 Q_2}{d^2}$$

An *electric field* is a region where an electric charge experiences a force.

The *electric field intensity,* E, at a point in an electric field is the force per unit positive charge at that point and its direction at that point is the same as the direction of the force. If a very small positive charge Q is placed at any point in an electric field and it experiences a force F, the field intensity at that point is defined by the equation,

$$E = \frac{F}{Q} \qquad \text{units of E are NC}^{-1}. \quad \text{(A commoner unit is the Vm}^{-1} \text{ - volt per metre)}$$

The electric field intensity due to a point charge Q, a distance d away from the point charge is given by

$$E = \frac{1}{4\pi\varepsilon} \cdot \frac{Q}{d^2}$$

The *potential difference* between two points in an electric field is the work done per unit positive charge (or the energy change per unit positive charge) passing from one point to the other. Unit of potential difference is the JC^{-1}, or the Volt V,

$$V = \frac{W}{Q} \qquad \textit{(Definition of Volt)}$$

If the potential difference between two points in an electric field is V volts, the energy change (or work done) occurring when a charge Q moves between the two points is, from the above definition, given by W = QV.

When an electron, whose charge is denoted by e, moves between two points in an electric field between which there is a p.d. of 1 volt, the work done (or energy converted) is W = eV (one electronvolt). This is a very small unit of energy. $1eV = 1.6 \times 10^{-19}$ J.

The charge storing ability, or *capacitance*, of an insulated conductor is defined as the charge required to cause unit change in the potential of the conductor. That is, if the potential of an insulated conductor changes by V, when give a charge Q, its capacitance C is

$$C = \frac{Q}{V}.$$

The capacitance of a parallel plate capacitor is given by

$$C = \frac{\varepsilon A}{d}.$$

The energy stored in a capacitor (work done in charging the capacitor) is given by
$$W = \tfrac{1}{2}CV^2.$$

The *electromotive force, emf*, of a source (a battery, generator, etc.) is the energy (chemical, mechanical, etc) converted into electrical energy when unit charge passes through it. (A voltage when applied to a circuit is called an emf.)

The *Ampere* is the constant current which, flowing in two infinitely long, straight, parallel conductors of negligible circular cross-section, placed in a vacuum 1 metre apart, produces between them a force of 2×10^{-7} newton per metre of their length.

One *Coulomb* is the quantity of electric charge carried past a point in a circuit when a steady current of 1 amp flows for 1 second. In general if a current I flows for a time t then the quantity of charge Q that passes is given by
$$Q = It.$$

The power of a device is the rate at which it converts energy from one form to another. If the pd across a device is V and the current flowing through it is I, then the electrical energy W converted by it to other forms (i.e. the work done) is (from W = QV)
$$W = ItV.$$

The Power of the device is

$$P = \frac{W}{t} = \frac{ItV}{t} = VI$$

If all the electrical energy is converted into heat the device is called a passive resistor and the rate of heat production will be VI. If the resistance of the resistor is R (the *resistance* of a conductor is <u>defined</u> as the ratio of the potential difference across it to the current flowing through it - unit is the Ohm Ω), then since V = IR

$$P = VI = IR.I = RI^2$$

Joule's Law states that the rate at which electrical energy is converted into heat energy in a resistor is proportional to the square of the current flowing,

$$P \propto I^2 \qquad (P = RI^2)$$

Ohm's law states that the resistance of a metallic conductor does not change with the p.d. across it, provided the temperature is constant. That is $V \propto I$. (If the temperature increases the resistance of the metallic conductor increases).

The resistance of a metallic conductor also varies with length and cross-sectional area - it is directly proportional to its length and inversely proportional to its cross-sectional area. This may be expressed as

$$R \propto \frac{l}{A} \quad \text{or} \quad R = \frac{\rho l}{A} \quad, \text{where } \rho \text{ is a constant called the } resistivity \text{ of the material of the conductor.}$$

The *resistivity* of a material can thus be defined as the resistance of a piece of the material of length 1 m and of cross-sectional area 1m^2, at a certain temperature. That is,
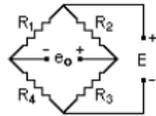
$$\rho = \frac{RA}{l}.$$

The total resistance R of several resistors in series is given by

$$R = R_1 + R_2 + R_3 + \ldots \ldots$$

The total resistance R of several resistors in parallel is given by

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots \ldots$$

In a Wheatstone bridge circuit, $R_1/R_2 = R_3/R_4$

The kilowatt-hour, kWh, is the unit of electrical energy used by the ESB. It is the amount of electrical energy converted into other forms by an appliance whose power is one kilowatt when switched on for one hour.

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A *magnetic field* is the space surrounding a magnet where a magnetic force is experienced.

In relation to the earth's magnetic field, the *angle of dip* at a point on the surface of the earth is the angle between the horizontal and the earth's magnetic field at that point. The *angle of declination* is the angle between the direction of magnetic north and geographical north.

A current carrying conductor is surrounded by a magnetic field and the right hand thumb rule gives the direction of the magnetic field - if the thumb of the right hand gives the direction of the current then the fingers give the direction of the field.

A current carrying conductor behaves like a magnet, so it experiences a force in a magnetic field. Fleming's left-hand rule gives the direction of the force experienced by a current carrying conductor in a magnetic field - If the first finger gives the direction of the magnetic field, the second finger the direction of the current, then the thumb will give the direction of the force experienced by the conductor.

The factors which affect the force on the conductor are current size, length of conductor in the magnetic field and the strength and direction of the magnetic field. Where θ is the angle between the conductor and the magnetic field, the force on the conductor is

$$F = BIl \sin\theta \qquad F = BIl \text{ when current flowing perpendicular to field}$$

The quantity which tells us how strong a magnetic field is we call the *magnetic flux density* and it is defined as the force acting per unit current length, i.e. the force acting per unit length on a conductor which carries unit current and is at right angles to the direction of the magnetic field. That is

$$B = \frac{F}{Il} \qquad \text{units are } N\,A^{-1}\,m^{-1} \text{ which is called the Tesla (T).}$$

The force on a particle of charge Q, moving with velocity v through a magnetic field (at an angle of θ to the direction of the magnetic field) of flux density B is

$$F = BQv \sin\theta \qquad F = BQv \text{ when particle moving perpendicular to field}$$

The production of electricity by magnetism is called *electromagnetic induction*. Faraday proposed that an e.m.f. is induced in a conductor when there is a change in the number of magnetic field lines 'linking' it or when it 'cuts' across field lines.

Magnetic field lines may be drawn so that the number per unit cross-sectional area represents the magnitude B of the flux density. The product BA, is taken as a measure of the number of field lines linking a coil of cross-sectional area A. This product is called the *magnetic flux* Φ, and is defined by

$$\Phi = BA \qquad \text{unit is } Tm^2 \text{ or weber, Wb.}$$

where B is the magnetic flux density acting at right angles to and over an area A.

Faraday's law of electromagnetic induction states the induced e.m.f. in a coil (of N turns) is directly proportional to the rate of change of magnetic flux linking the coil. That is,

$$E = \frac{d}{dt}(N\Phi) \qquad \text{(constant of proportionality is 1)}$$

Lenz's law of electromagnetic induction states that the direction of the induced e.m.f. is such that it tends to oppose the flux change causing it, and does oppose it if induced current flows. That is,

$$E = -\frac{d}{dt}(N\Phi) \qquad (\text{ - sign incorporates Lenz's law})$$

The r.m.s. value of an alternating voltage, V, is related to its peak value V_0 by the formula $\qquad V = \frac{V_0}{\sqrt{2}}$

The r.m.s. value of an alternating current, I, is related to its peak value I_0 by the formula $\qquad I = \frac{I_0}{\sqrt{2}}$

When an alternating p.d. is applied to the primary coil of a transformer , the resulting current produces a large alternating magnetic flux which links the secondary and induces an e.m.f. in it. The value of this e.m.f. is given by

$$\frac{\text{e.m.f. induced in the secondary}}{\text{p.d. applied to the primary}} = \frac{\text{No. of secondary turns}}{\text{No. of primary turns}}$$

If the transformer is 100% efficient, then, considering the law of conservation of energy we can say that the power in the primary must equal the power in the secondary. Sine P = VI, we have $V_1I_1 = V_2 I_2$.